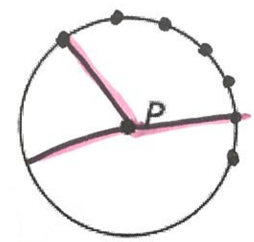
Date: Period: 3

Lines and Segments that Intersect Circles

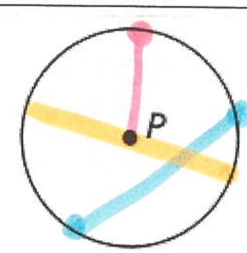
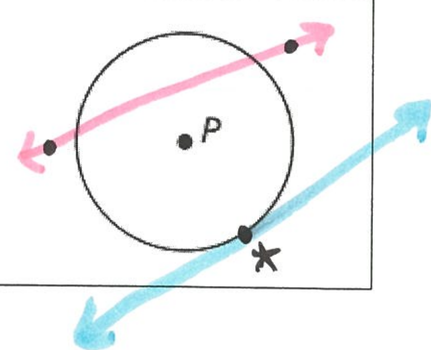
Lesson Objective IWBAT identify parts of a ⊙; use properties of tangent segments; constructions w/ ⊙s

Circle: A circle is the set of all points in a plane that are equidistant from a given point called the center of the circle.

radius

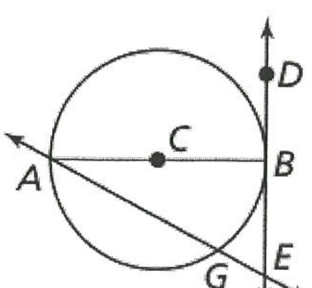


➔ A circle with center *P* is called "circle *P*" and can be written as ⊙P

Vocab Term	Definition	Picture
Radius	segment whose endpoints are the center and ~~any~~ any point on ⊙	
Chord	segment whose endpoints are on the circle	
Diameter	a chord that contains the center of the circle	
Secant	a line that intersects a circle in 2 points	
Tangent	a line in the plane of a circle that intersects the circle at exactly one point	
Point of Tangency *	where the tangent intersects the circle	

Example

1. Tell whether the line, ray, or segment is best described as a *radius, chord, diameter, secant,* or *tangent* of ⊙ *C*



a. $\overline{AC}$ radius

b. $\overline{AB}$ diameter

c. $\overrightarrow{DE}$ tangent ray

d. $\overleftrightarrow{AE}$ secant

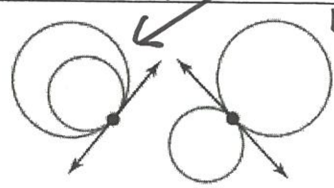
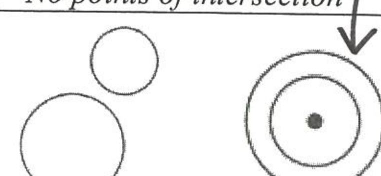
e. $\overline{AG}$ chord

tangent circles

concentric circles

Coplanar Circles and Common Tangents

In a plane, two circles can intersect in two points, one point, or no points.

Two points of intersection	*One point of intersection*	*No points of intersection*
		

Tangent Circles: Coplanar circles that intersect in one point are called tangent circles

Concentric Circles: Two circles that have a common center are called concentric circles

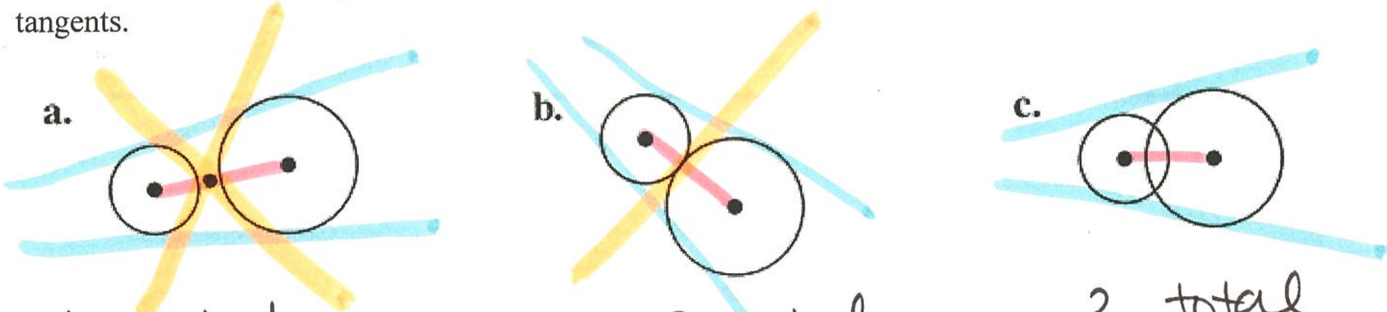
Common Tangent: A line or segment that is tangent to two coplanar circles is called a common tangent.

→ A common internal tangent intersects the segment that joins the centers of two circles

→ A common external tangent do not intersect the segment that joins the centers of two circles

Example:

2. Use one color to draw any common external tangents, and a different color to draw any common internal tangents.



a. 4 total common tangents

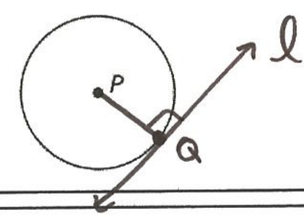
b. 3 total c.t.

c. 2 total c.t.

Tangent Line to Circle Theorem

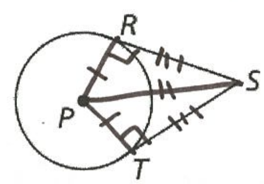
In a plane, a line is tangent to a circle if and only if the line is ⊥ to a radius of the circle at its endpoint on the circle.

line $l \perp \overline{QP}$



External Tangent Congruence Theorem

Tangent segments from the same external point are congruent.



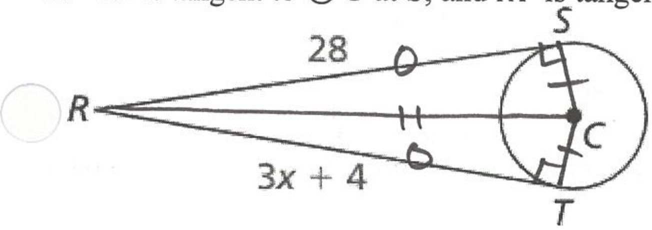
PROOF: **Given**: $\overline{SR}$ and $\overline{ST}$ are tangent to ⊙ P

Prove: $\overline{SR} \cong \overline{ST}$

Statements	Reasons
① $\overline{SR}$ and $\overline{ST}$ are tangent to ⊙P	① given
② Draw $\overline{PR}$, $\overline{PT}$, $\overline{PS}$	② Thru any 2 points there is one line
③ $\overline{PR} \perp \overline{SR}$, $\overline{PT} \perp \overline{ST}$	③ Tangent Line to ⊙ Thm
④ ∠PRS, ∠PTS are right ∠s	④ Def. of ⊥ lines/segs.
⑤ △PRS, △PTS are right △s	⑤ Def. of a right △
⑥ $\overline{PS} \cong \overline{PS}$	⑥ reflex. prop. ≅
⑦ $\overline{PR} \cong \overline{PT}$	⑦ radii in same ⊙ are ≅
⑧ △PRS ≅ △PTS	⑧ HL ≅
⑨ $\overline{SR} \cong \overline{ST}$	⑨ CPCTC

Example:

3. $\overline{RS}$ is tangent to $\odot C$ at S, and $\overline{RT}$ is tangent to $\odot C$ at T. Find the value of x.



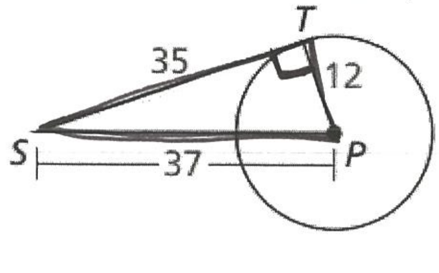
$3x+4=28$

$3x=24$

$\boxed{x=8}$

if $a^2+b^2 \neq c^2$
then no rt. ∠
then no
tangent

4. Is $\overline{ST}$ tangent to $\odot P$? → is $\overline{ST} \perp \overline{TP}$?



if $a^2+b^2=c^2$ → rt. Δ

$12^2+35^2=37^2$

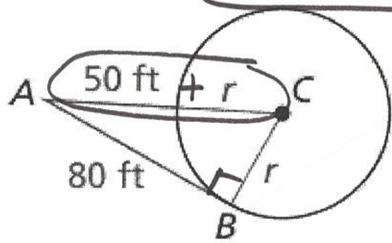
$1369=1369$

ΔSTP is a rt. Δ w/ rt. ∠ @ ∠T. So, $\overline{ST} \perp \overline{TP}$

so, $\overline{ST}$ is tangent to ⊙P @ T.

5. In the diagram, point B is a point of tangency. Find the radius r of $\odot C$.

↳ $\overline{AB} \perp \overline{BC}$



$r^2+80^2=(50+r)^2$ → $\neq 50^2+r^2$ BAD! ⩉

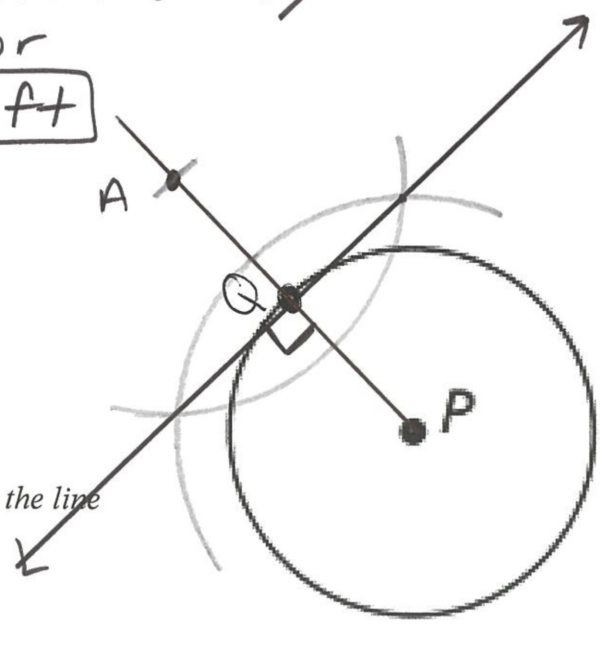
$r^2+6400=(50+r)(50+r)$

$\cancel{r^2}+6400=2500+100r+\cancel{r^2}$

$3900=100r$

$\boxed{r=39 \text{ ft}}$

Construction #1: Tangent to a circle at a point on the circle



1. Draw the ray that extends from the center of the circle through the point of tangency.
2. Construct a line perpendicular to the ray from step #1 that goes through the point on the circle
   a. This is a construction from first semester!
      Construct the perpendicular line through a point on the line

Construction #2: Tangent to a circle through a point outside the circle

1. Draw the segment whose endpoints are the center of the circle and the point outside the circle
2. Construct the midpoint of the segment from step #1. Label this point.
3. Construct the circle whose center is the midpoint from step #2, and goes through the endpoints of the segment from step #1.
    a. This circle will intersect the original circle twice. Label one of these points.
4. Draw the line that goes through the intersection point from step #3 and the original point outside the circle.
    a. This line is tangent to the original circle!